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# On the Use of Experimental Design in Metamodling of Analog Integrated Circuits

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# On the Use of Experimental Design in Metamodeling

# Of Analog Integrated Circuits

By

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# DECLARATION

I am Huda Al-khateeb, I hereby declare that this thesis, which I submit to the Department of Computer Engineering at Hijjawi Faculty for Engineering Technology, is my own work, and I have not plagiarized from any sources. All references of sources are given and cited in my thesis. Each significant contribution to and quotation in this report from work of other people has been attributed and referenced.

*Huda Al-khateeb*

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# SYMBOLS AND ABBREVIATIONS

| | |
|---|---|
| **NRMSE……………………………………** | Normalized root mean square error |
| **RSM…………………………………** | Response surface model |
| **LHC…………………………………** | Latin hypercube |
| **DOE…………………………………** | Design of experiment |
| **MBD…………………………………** | Minimum bias design |
| **CMRR………………………………** | Common mode rejection ratio |
| **ACO…………………………………** | Ant Colony Optimization |
| **SVM…………………………………** | Support vector machine |
| **RMSE………………………………..** | Root mean square error |
| **IC……………………………………** | Integrated circuit |
| **BW…………………………………..** | Band width |
| **PW…………………………………..** | Power dissipation |
| ***A*……………………………………..** | Midband gain |
| $f_l$…………………………………….. | Lower band cutoff frequency |
| $f_H$…………………………………… | Higher band cutoff frequency |

# ABSTRACT

**Al-Khateeb, Huda Turki. On the Use of Experimental Design in Metamodling of Analog Integrated Circuits. MSc. Thesis, Yarmouk University, 2017. (Supervisor: Dr. Sami Al-Hamdan, Co-supervisor: Prof. Husam Hamad)**

A metamodel or surrogate model is a model of a model. The process of generating such metamodels is called metamodeling. A model is an abstraction of a phenomenon in the real world such as computer simulators. Thus, a metamodel is a simplified abstraction of the original model that makes complex computer simulations of the original model simpler, yet faster while keeping acceptable accuracy. Metamodels have become widely used across engineering and science disciplines. A successful metamodel requires careful choosing of appropriate experimental designs. An experimental design is a set of design variable values (inputs) that are used to generate a metamodel for a response as a function of the design variables. Analog electronic circuit optimization is usually a computationally intensive problem. The use of metamodeling in this class of problems to replace simulators is a promising technique for time reduction of circuit optimization. One of the criteria that the metamodeling techniques need to be tested on is accuracy. Accuracy is largely dependent on the type of the metamodel and the computer experimental design used to generate the metamodel. In this research, two of the most popular metamodeling types are investigated: the classical response surface models (RSMs), and the more recent Kriging metamodels, with two main computer experimental design methods: Latin hypercube (LHC) sampling and minimum bias designs (MBDs). Each method is applied in this work to model circuit performance parameters of analog electronic circuits, ranging from simple electronic filters to analog integrated circuits such as operational amplifiers. The results of this investigation show that the use of RSMs metamodels combined with MBDs sampling is superior to the more popular Kriging technique combined with LHC sampling. This work should direct the analog integrated circuit design research community to the most suitable methodology in both metamodel types and experimental designs to use.

Keywords: Metamodel, Kriging, Response Surface Model (RSM), Minimum bias design (MBD), Latin hypercube (LHC), Root mean square error (RMSE).

# CHABTER I: INTRODUCTION

## 1.1 Introduction to metamodeling and experimental designs

Traditional simulation methods used in computer-aided design (CAD) of analog integrated circuits (ICs) is time consuming, especially if the number of active components is large [1]. The number of transistors per unit area is on the rise; hence, the design and manufacture of these ICs take long time. Computer simulations are usually involved, and the complexity of the circuit determines the time needed to design the circuit. It may take from a few seconds or minuets to several hours or days [2]. Metamodels (surrogate models) of circuit performance parameters are widely used instead of simulators to reduce the time of circuit design optimization, and hence allow faster production of analog ICs.

A metamodel is an approximation of a physical/electrical model which approximates as closely as possible the original model; e.g., to characterize the performance parameters of a circuit [1]. In other words, it is a mathematical model that uses samples to generate a function that estimates the relation between the design variables and the performance parameters of the circuit under investigation [2]. These approximation models are used in the design process to reduce the design cycle time and cost by predicting the output of an expensive computer code at many points in the design variables space [3].

Different types of metamodels are used in the literature to model analog integrated circuits performance parameters such as Kriging [1, 4, and 5]; Response Surface Models (RSMs) [6], Radial Bases Functions (RBF) [4], rational functions [4] and support vector machines (SVM) [2].

Metamodels have to be calibrated to achieve operable accuracy levels and to deal with specific user requirements. Computer experimental designs are used to determine combinations of design variables to fit these metamodels. Different experimental designs to build metamodels are used in the literature including Latin hypercube (LHC) sampling [1, 4], factorial designs [5], Box-Behnken designs [5] and minimum bias designs (MBDs) [6].

The statistics given in the next section indicate more and more researches combine Kriging metamodels with LHC sampling to build metamodels for circuit performance parameters. It is widely believed that this combination requires less sample points and provides more accuracy than other metamodels.

In this work, we investigate if the increasing popularity for using Kriging metamodels in analog ICs design is justified in terms of accuracy, by comparison to the classical RSM metamodels. However, unlike most recent work that derives RSM metamodels using LHCs, MBs are used in this work to build RSMs.

## 1.2 Research Statistics

Figure (1.1) summarizes Google Scholar search results showing the number of research papers that use Kriging and RSM metamodels for the period (2000-2015), inclusively.

The following statistics are deduced from the results in Figure (1.1):

- For Kriging metamodels, the number of articles that mention Kriging metamodels has increased from 321 articles during (2000-2007) to 1340 articles for (2008-2015), i.e., the increase is by 3.2 folds.

- For RSM metamodels, the number of articles that referred to RSM metamodels has increased from 196 articles during (2000-2007) to 425 articles for (2008-2105) i.e., the increase is by 0.17 folds.
- In (2000-2007), 62% of articles on electronic circuits relate to Kriging metamodels by comparison to RSM metamodels, while this has increased to 76% for the period (2008-2015).

The above results are summarized in Figure (1.2). Based on these statistics, Kriging metamodels are more popular than RSM in articles concerning metamodeling in electronic circuits. Moreover, this popularity is on the rise.

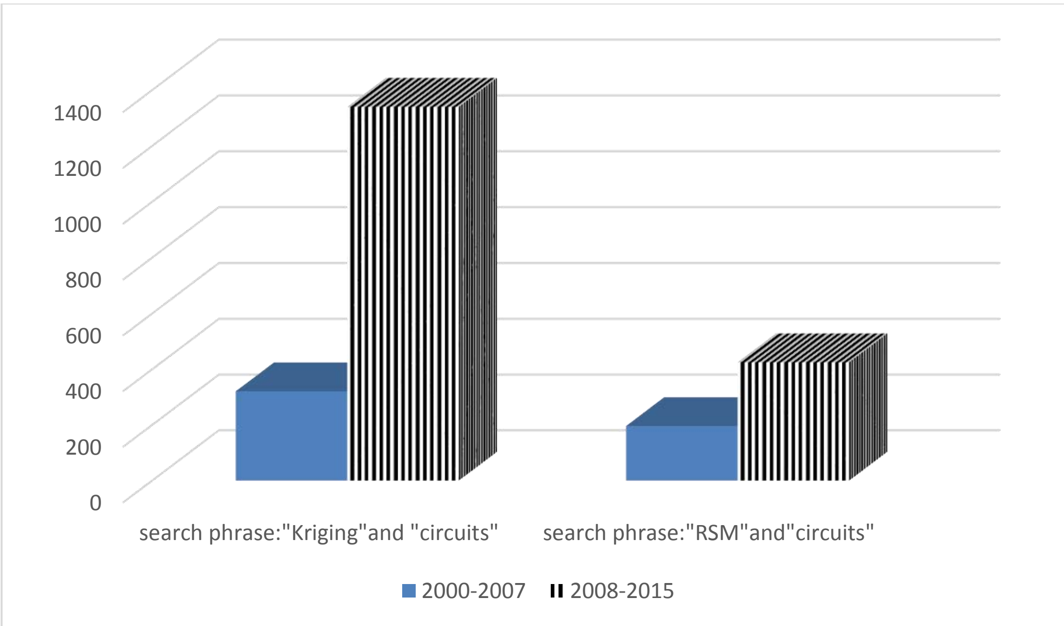


Figure 1.1: Google search results for the number of articles on electronic circuits using Kriging and RSM metamodels for the period 2000-2015.


Figure 1.2: percentage of articles on electronic circuits using Kriging and RSM metamodels in two different time periods.

A Similar Google Scholar search is conducted for LHC and MBD experimental design methods, with the results summarized in Figure (1.3) for the period (2007-2015), inclusively.

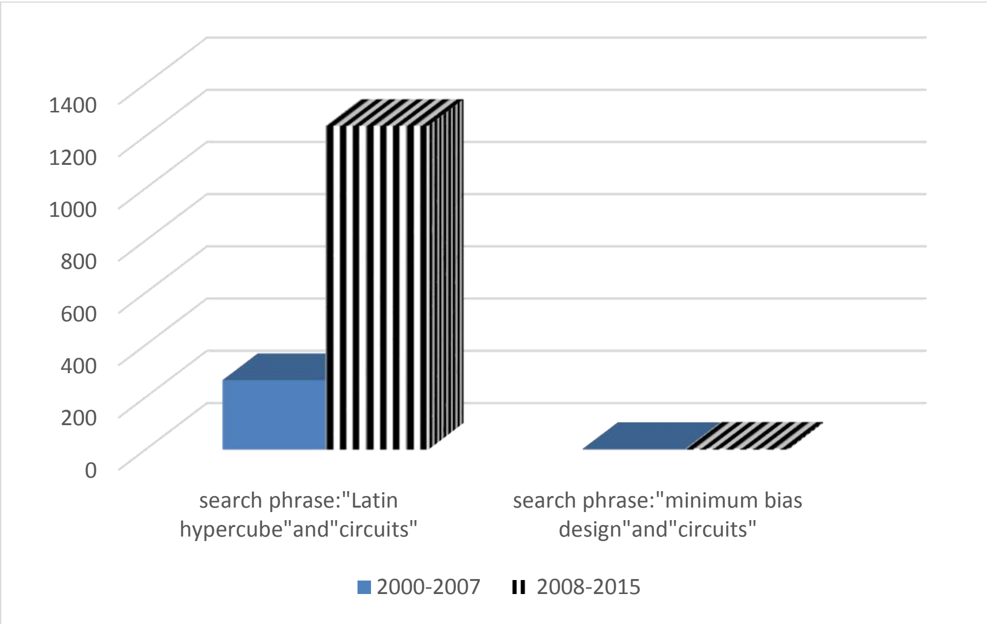


Figure 1.3: Google search results for the number of articles on electronic circuits using LHC and MBD experimental designs for the period 2000-2015.

The following statistics based on the results in Figure (1.3) indicate that:

- For the LHC sampling method, the number of articles that mention LHC experimental design has increased from 266 articles during (2000-2007) to 1240 articles for (2008-2015), i.e., the increase is by 3.6 folds.
- For the MBD sampling method, the number of articles that mention MBD experimental design has increased from 3 articles during (2000-2007) to 4 articles for (2008-2015), i.e., the increase is by 0.33 folds.
- In (2000-2007), 98.8% of articles on electronic circuits refer to LHC sampling method by comparison to MBD sampling method, while this has increased to 99.7% for the period (2008-2015).

In summary, it is clear from the research statistics presented in this section that Kriging metamodeling and LHC sampling are much more widely used in the literature on electronic circuits by comparison to the classical RSMs and MBDs. More discussion about this issue is presented in Chapter 3.

## 1.3 Research contribution

In the design of analog integrated circuits, analysis to determine the required circuit performance parameters is executed using time-consuming simulations. A lot of time is consumed to determine and satisfy circuit performance parameters at different conditions. This analysis should be performed rapidly because there are some strict time-to-market restrictions in the industrial sector [1, 2].

For these reasons, the use of metamodels (surrogate models) has become an effective technique for estimating the behavior of a circuit with high accuracy. Using suitable metamodels instead of the traditional simulation tools gives acceptable results in much shorter periods [1, 2].

The objective of this work is to shed light on the proper directions in metamodeling techniques and experimental design methods for analog integrated circuits and their application in the design of such circuits. In particular, we investigate if the increasing popularity for using Kriging metamodels with LHC sampling in electronic circuit design is justified in terms of accuracy by comparison to RSM metamodels with MBD sampling.

## 1.4 Structure of the thesis

This thesis is divided into six chapters. An outline of the remaining chapters is as follows:

- Chapter 2 summarizes results in recent published research in the area of metamodeling, especially in analog integrated circuit design.
- Experimental design methodologies are presented in Chapter 3, which includes MATLAB codes to generate LHC and MBD designs, and discusses issues related to metamodel types, with emphasis on analytical formulation of Kriging and RSM metamodels.
- Analog circuits ranging from simple filters to the most commonly used analog IC (the operational amplifier) are presented in Chapter4, comparing Kriging with LHC experimental design and RSM with MBDs.
- Chapter 5 concludes the thesis, giving directions for future work.

# CHAPTER II: LITERATURE REVIEW

Metamodels are not new; they have been in use for more than sixty years [7, 8]. Most commonly in recent literature, metamodels are used to replace simulators in the design of engineering systems, as a way to minimize the time used for estimation of system performance parameters.

Traditionally, the default metamodeling technique is polynomial regression metamodels [7, 8]; nowadays more and more different techniques of metamodeling are used to reduce computation time for engineering system design.

Different types of metamodels are used in analog circuit design. These types include Kriging, Response Surface Models (RSMs), Radial Bases Functions, Rational Functions, Support Vector Machines (SVM), etc. In addition, different experimental designs to build metamodels of integrated circuit performance parameters are used; these include simple random sampling, classical sampling, LHCs, MBDs, etc.

H. You [1] explored the attributes of combining classical Kriging metamodels with LHC sampling method, and RSM metamodels with classical Design of Experiment (DOE) sampling methods. In this reference, two circuit performance parameters of an amplifier are characterized: power dissipation (Pc) and bandwidth (BW). The result shows that the Kriging metamodel with LHC sampling method needs less sample points and provides higher accuracy than quadratic RSM metamodels with classical Design of Experiment ( DOE) sampling methods.

A. Ciccazzo, et al in [2] compared between two metamodels, support vector machines (SVM) and RSMs. These metamodels are compared on WiCkeD tool, and the conclusion was that the two metamodels give quite similar results.

H. Zhong-Hue, and K. Zhang, in [3] focuses on exploring the differences between the two-metamodeling techniques: Kriging and polynomial regression. The goal is to discover which technique is most suitable. Regarding the use of experimental design, LHC sampling and random design sampling are used to test the performance of each metamodeling technique with four different simulation models. The overall results show that Kriging metamodeling has a better performance of speed and accuracy on average than regression metamodeling.

In [4], Kriging, radial bases functions (RBF), and rational functions metamodels combined with LHC were applied to model performance parameters of a transimpedance amplifier circuit. The authors compared results of these three metamodeling types to find an optimal solution in a short design time. Three circuit performance parameters are characterized for the transimpedance amplifier: bandwidth (BW), gain (Zg), and power consumption (pwr). Efficiency for these metamodels are compared, the results show that the three metamodels are very fast, they take a few seconds to optimize the circuit, but the rational functions metamodels with LHC was the fastest one.

H. Hamad, and A. Bani Irshaid in [5] constructs a piecewise –Kriging metamodel to reduce the complexity of variation of the design variables space and thus enhancing accuracy of the metamodels, by dividing design variables space into several pieces. Two metamodeling techniques Global-Kriging and piecewise-Kriging metamodels combined with LHC methods are compared, showing that more accurate metamodels can be achieved using piecewise- Kriging metamodels.

H. Hamad, et al in [6] presented techniques for constructing minimum bias response surface metamodels for deterministic simulation models using minimum bias designs (MBDs). This technique can be used to build MBDs for higher-dimension spaces and higher-order MBDs. Analytic examples are used in this work to demonstrate that the MBDs sampling combined with RSMs metamodels are potentially superior to the more popular LHCs sampling combined with RSM metamodels.

A new methodology is presented in [9] that combines the Kriging technique with Ant Colony Optimization (ACO) algorithm that presents fast optimization of the circuit. This methodology is examined using an amplifier integrated circuit. The result is that the Kriging based metamodel is accurate, and (ACO) algorithm improves sense amplifier precharge time to 3.7 minutes compared with 72 hours.

D. Gorissen, et al in [10] made a comparison of accuracy and scalability of different metamodels including artificial neural network model (ANN), Kriging, SVM and rational function models, based on samples for a low noise amplifier LNA of RF circuit block. It was clear that the ANN models gave excellent results with SVM functions compared to other metamodels.

An adaptive RSM-based optimization method for analog circuit sizing is presented in [11] to reduce the computational cost of designing applications requiring computationally expensive evaluations. Through application to different test functions and case studies including a two-stage operational amplifier, the method demonstrated effectiveness compared to annealing and differential evolution technique. The result was clear that the ANN models perform best in Kriging

In [16], as a comparative case study two circuits have been designed : a 180nm LC-VCO and a 45 nm ring oscillator (RO), to investigate three algorithms to compare the speed of optimization on polynomial metamodels. The results show that metamodel-based optimization achieved speed up as high as 21,600× for the LC-VCO circuit and 11,750× for the RO compared to the actual circuit netlist-based (SPICE) optimization.

O. Garitselov, et al in [30] presented a two-tier approach to reduce the design cycle time by combining accurate metamodeling and intelligent optimization. This paper introduced an intelligent Bee Colony Optimization (BCO) algorithm to speed up the design-space exploration for AMS circuits. A 180 nm LC-VCO PLL frequency generation circuit is used as case study, the result of the design flow was 90% power savings and average of 52% jitter minimization, which have been achieved with a minimal time of 100 simulations to generate polynomial metamodels.

# CHAPTER III: METAMODELING AND EXPERIMNTAL DESIGN

As has been emphasized in this thesis so far, a metamodel is an interpolation function that represents the relation between design variables (inputs) and system performance parameters (outputs) of a simulation model to estimate the behavior of a system as a ***black box*** model. These metamodels are used instead of the more time consuming simulators to reduce the time of system design optimization [12]. The goal of this chapter is to present the different metamodel types and methodologies, including experimental design techniques. The chapter starts by outlining the metamodeling steps.

## 3.1 Metamodeling steps

Metamodeling consists of four steps: sampling, fitting, reproducing, and validation; see Figure (3.1).

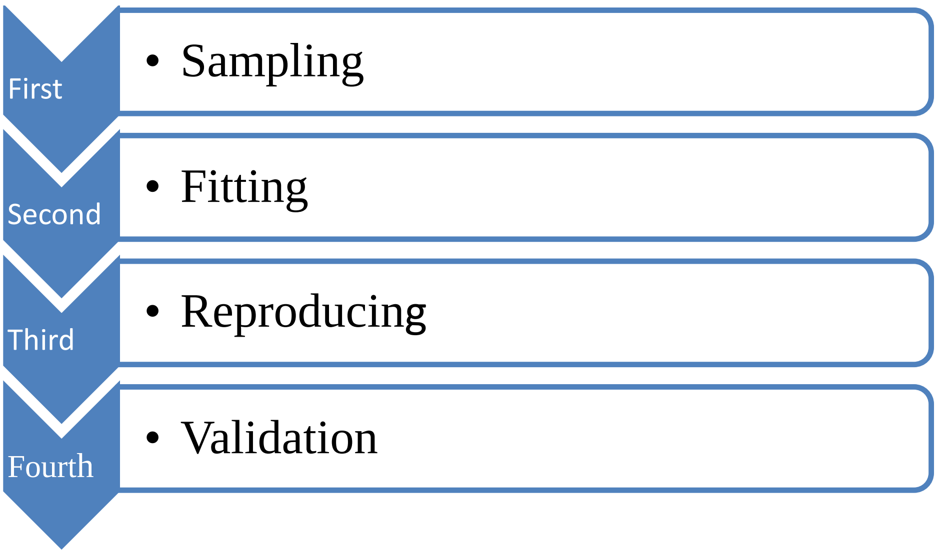


Figure 3.1: Metamodeling steps.

- **Sampling:**

  Sampling is the way of selecting a pre-determined small number of points in the design variables space to be used for generating the interpolation function (metamodel). Choosing the appropriate DOE (i.e. the set of samples) is very important for accurate metamodel results.

- **Fitting:**

  In this phase, a metamodel is fitted to the set of sample points chosen in the previous step. The parameters of the chosen metamodel type are adjusted to minimize error (usually least-square-error).

- **Reproducing:**

  In this phase, the parameters of the metamodel computed in the previous step are used to give estimation of the response at a wider set of points in the design variables space.

- **Validation:**

  In this phase, the accuracy of the chosen metamodel is tested against the original model used usually by a simulator (e.g. SPICE). Accuracy is one of the criteria that metamodeling techniques are tested on. It is a predictive quality of a metamodel, i.e. goodness-of-fit between the metamodel and the response. In statistics, various validation measures exist, e.g., coefficient-of-determination $R^2$, mean absolute error (MAE), and root mean square error (RMSE) [8]. RMSE - the most popular validation statistic - is used in this research for metamodel validation.

RMSE is a measure of the difference between the response predicted by the metamodel and that used by the actual model in simulators [13]. RMSE is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2}{n}} \quad \text{.......................................................(1)}$$

Where $Y_i$ is the actual response at the $i_{th}$ sample point, $\hat{Y}_i$ is the predicted response by the metamodel at the same point [12].

To reflect a percentage-wise error, normalization may be used to determine normalized error NRMSE, e.g.

$$NRMSE = \frac{RMSE}{|Y_{max} - Y_{min}|} \quad \text{.......................................................................(2)}$$

$Y_{max}$, and $Y_{min}$ are the maximum and minimum values of the response respectively.

Experimental design sampling methods and metamodel types are briefly discussed in the next sections. For reasons presented in Chapter I, emphasis will be placed on two metamodel types-Kriging and RSM metamodels, and on two experimental design methods-LHC sampling and MBDs sampling.

## 3.2 Experimental design methods

When selecting a metamodel technique, probably the most important issue that has to be taken into account is the experimental design (the sampling method) of the design variables space. An experimental design is a software structure that assist users with designing and exploring experiments and their results, and it involves selecting the right variations in input design variables to build a model of the performance (response) as a function of these design variables [8].

In the designing process, exploring all area of the of the design variables space requires high cost of performing many experiments and it takes long time. Instead, an experimental design is used to determine the location of a set of sample points in the design variables space that covers the information gained from the response that is necessary to fit the metamodel [15].

Currently, there are two different types of experimental designs: ‘classic’ experimental designs, and ‘modern’ experimental designs. Classic experimental designs such as factorial designs, Box-Behnken designs, and composite central designs [5] are traditionally used in response surface models, while modern experimental designs such as LHC [16, 17] and Orthogonal Array Design (OAD) [5] are mostly used with Kriging metamodels. In this research, two experimental designs are investigated: LHC and MBD sampling.

### 3.2.1 Latin hypercube sampling method

Latin hypercube (LHC) sampling is a type of stratified sampling (sampling from a population). It works by controlling the way that random samples are generated for a probability distribution [16, 17]. An LHC sample is generated by dividing the design variables space into subintervals and choosing randomly a sample in these subintervals, and ensuring that every design variable is used exactly once, where each sample covers one of possible probabilities of all design variables, for example in Figure (3.2) one square in each row and column contains one sample chosen randomly in the space covered by that square [3].

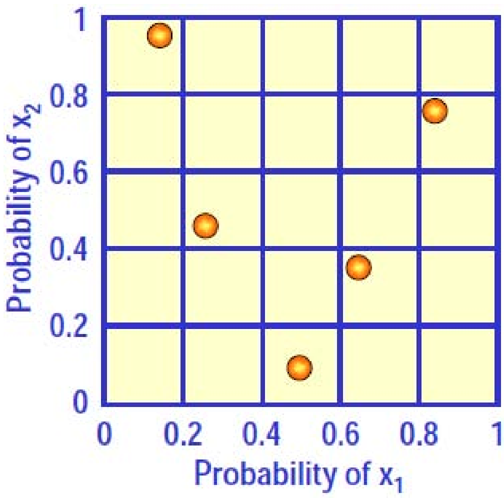


Figure 3.2: Two-dimensional LHC [2].

There are several combinations to choose the sampling points while preserving the conditions imposed on LHC method. The maximum number of combinations for a Latin hypercube of *M* divisions and N design variables (i.e., dimensions) can be computed with the following formula [16]:

$$\left(\prod_{n=0}^{M-1}(M-n)\right)^{N-1} = (M!)^{N-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

For example, a Latin Hypercube with $M = 4$ divisions and $N = 2$ variables has 24 possible combinations.

In this work, LHC sampling is generated using command "*lhsdesign*" in MATLAB software. This command requires a prior knowledge about the number of sample points to be generated and the number of design variables. It generates sample points with coordinates of values between 0 and 1; this is called the "coded" variable. The coded variables are then converted to their true "natural" values of the design variables. For example: a resistor in an electronic circuit that could have values from 200 Ω to 1000 Ω is mapped to values from 0 to 1 in LHC such that 0 in LHC will be the corresponding value for 200 and 1 will map to 1000. Generally, the natural values for a variable x are computed as follows:

$$xn = xc\,(xn_{max} - xn_{min}) + xc_{min} \quad \ldots\ldots(4)$$

Where xn and xc are the natural and coded values of the variable x respectively.

### 3.2.2 Minimum bias designs

When finding an interpolation function using a chosen set of points from the design variable space, the resulting function will have errors due to two major factors: variance error which is primarily caused by sampling, and bias error which is caused by the choice of the interpolating function (e.g. choosing a first order polynomial for a metamodel while the actual model is of second order nature) [6]. MBD is concerned with the selection of sample points from the design variable space such that the error caused by the wrong choice of the metamodel function is minimized.

Box and Draper introduced the minimum bias (all bias) criterion to generate MBDs [19], with more recent treatment by [6] and [29]. Here statistical design of experiments is used to select optimal points that minimize bias error in the metamodel approximation. Unlike classical designs which minimize variance error assuming no bias in the metamodel (i.e., the metamodel perfectly matches the complexity of the underlying response), MBDs automatically minimize the bias error and the mean square error.

The following MBD derivation outline parallels that in [6]:

Let Y(X) be a function in *k* variables $x_1, x_2 \ldots x_k$ and $\hat{Y}(B1, X)$ is an approximation over the region R in the design variables space. Assuming $\hat{Y}(B1, X)$ is a polynomial of degree $d_1$, where

$$\hat{Y}(B_1, X1) = B_1X1^T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

X1 is the vector of m elements of powers and products of $x_1$, $x_2$, ...$x_k$ of order $d_1$ or less, and B1 is the corresponding $m1$ coefficients, where

$$m1 = \frac{(d1 + k)!}{d1!\,k!} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Suppose the true function Y(X) is represented be a Weierstrass polynomial $\breve{Y}(B, X)$ of degree $d_2 > d_1$ , where

$$\breve{\boldsymbol{Y}}(\boldsymbol{B}, \boldsymbol{X}) = \boldsymbol{B_1X1^T} + \boldsymbol{B_2X2^T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

***X1*** is as before, ***B₁*** is the corresponding vector of coefficients, ***X2*** has $m_2$ elements of powers and products of $x_1$, $x_2$, ..., $x_k$ of order $d_2$ or less but greater than $d_1$, and $\boldsymbol{B_2}$ is the corresponding vector of $m_2$ coefficients, where

$$m2 = \left(\frac{(d2 + k)!}{d2!\,k!}\right) - m1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

[19] uses equation (5) to minimize the average integrated bias–*AIB*

$$AIB = \frac{1}{V}\int_R \left[\breve{\boldsymbol{Y}}(\boldsymbol{B}, \boldsymbol{X}) - \hat{\boldsymbol{Y}}(\boldsymbol{B_1}, \boldsymbol{X1})\right]^2 dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

Where

$$V = \int_R dx \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Then a sufficient condition to minimize $AIB$ is

$M_{11}= \mu_{11}$ , $M_{12} = \mu_{12}$ .................................................................................... (11)

$\mu_{11}$ and $\mu_{12}$ are known as the moments of the region *R*. The $m_1$ x $m_1$ elements of $\mu_{11}$and $m_1$ x $m_2$ elements of $\mu_{12}$ are of the form

$$\frac{1}{V}\int_R x_1^{a_1}\, x_2^{a_2} \ldots x_k^{a_k}\, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

These moments are said to be of order $a; a = a_1 + a_2 + \cdots + a_k$ and $1 \leq a \leq (d1+ d2)$. Similarly, for the *N* experimental design points, the ($m_1$ x $m_1$) moments $M_{11}$ and ($m_1$x$m_2$) moments $M_{12}$ of order $a; a = a_1 + a_2 + \cdots + a_k$ have elements of the form:

$$\frac{1}{N}\sum_{n=1}^{N} x_{1n}^{a_1}\, x_{2n}^{a_2} \ldots x_{kn}^{a_k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

It can be shown that the number of moments $n_m$ of order $a(1 \leq a \leq (d1+ d2))$ which must be equalized in accordance with equation (11) to form a MBD in an arbitrarily shaped *k*-dimensional region *R* is

$$n_m = \sum_{a=1}^{d1+d2} \frac{\big(a + (k-1)\big)!}{a!\,(k-1)!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14) .$$

Table (3.1) shows $n_m$ values for second to fourth-order MBDs for $2 \leq k \leq 5$.

Table3.1: Number of moments in equation (14)

| *MBD* | | | |
|---|---|---|---|
| *k* | 2nd order | 3rd order | 4th order |
| 2 | 20 | 35 | 54 |
| 3 | 55 | 119 | 219 |
| 4 | 125 | 329 | 714 |
| 5 | 251 | 791 | 2001 |

Examining the table above reveals that the number $n_m$ of nonlinear equations that must be satisfied can be prohibitive for large dimensions and high-order MBDs. Fortunately, the number of such equations can be reduced dramatically by:

- Using symmetrical regions R. For example, for a *k*-dimensional cuboidal region with $-1 \leq x_1, x_2, \ldots x_k \leq +1$, (here, the range -1 to +1 is the coded values for all variables of the system space) the integration of eq. 12 above reduces to zero for all odd powers:

$$\frac{1}{V}\int_R x_1^{a_1} x_2^{a_2} \ldots x_k^{a_k}\, dx = \begin{cases} 0 & if any\ a_i\ is\ odd \\ \dfrac{1}{(a_1+1)(a_2+1)\ldots(a_k+1)} & all\ a_i\ even \end{cases}$$

To illustrate, suppose it is required to generate a second-order MBD in a two-dimensional region R. From Table 1, the number of moments that must be equalized in this case is 20. If no symmetry is used, then the coordinates $(x_1, x_2) = (a_i, b_i)$ for $1 \leq i \leq 10$ are determined by solving the following 20 × 20 system of the nonlinear equations:

- First-order moments:

$$\sum_{i=1}^{10} a_i = N\int x_1 dx$$
$$\sum_{i=1}^{10} b_i = N\int x_2 dx \qquad \ldots\ldots\ldots\ldots\ldots\ldots(15)$$

- Second-order moments:

$$\sum_{i=1}^{10} a_i^2 = N\int x_1^2 dx$$

$$\sum_{i=1}^{10} a_i b_i = N \int x_1 x_2 dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(16)}$$

$$\sum_{i=1}^{10} {b_i}^2 = N \int {x_2}^2 dx$$

- Third-order moments:

.

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- Fourth-order moments:

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- Fifth-order moments:

$$\sum_{i=1}^{10} {a_i}^5 = N \int {x_1}^5 dx$$

$$\sum_{i=1}^{10} a_i \, {b_i}^4 = N \int x_1 {x_2}^4 dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(17)}$$

.

.

.

$$\sum_{i=1}^{10} {b_i}^5 = N \int {x_2}^5 dx$$

The complexity of the above system reduces to the solution of the following 3 × 3 nonlinear system if the symmetrical point sets are used in the 'cuboidal' space $-1 \leq x_1$ ,$x_2$ $\leq +1$:

$$4a^2 + 4b^2 = \frac{n_0 + 8}{3}$$

$$8a^2b^2 = \frac{n_0 + 8}{9} \qquad \text{........................................(18)}$$

$$4a^4 + 4b^4 = \frac{n_0 + 8}{5}$$

Where $n_0$ is the number of center points $(x_1, x_2) = (0, 0)$.

Table 3.2: Notation used.

| *Notation* | *Meaning* | *#points* | *Notes* |
|---|---|---|---|
| $C(0^k)$ | A design point at the centre | 1 | $x_1 = \cdots x_k = 0$ |
| $F(u^k)$ | Factorial design | $2^k$ | See Table 3.3 for $k = 3$ |
| $S(u^k, v^{k-a})$ | All $k$ permutations of factorial designs with $a$ variables at $u$ and $(k–a)$ variables at $v$ | $k2^k$ | See Table 3.4 for $k = 3$ |

Table 3.3: $F(u^k)$ factorial design for $k = 3$

| $x1 = \pm u$ | $x2 = \pm u$ | $x3 = \pm u$ |
|---|---|---|
| -*u* | -*u* | -*u* |
| -*u* | -*u* | +*u* |
| -*u* | +*u* | -*u* |
| -*u* | +*u* | +*u* |
| +*u* | -*u* | -*u* |
| +*u* | -*u* | +*u* |
| +*u* | +*u* | -*u* |
| +*u* | +*u* | +*u* |

The point sets in Draper (1960) which are used to form rotatable designs in cuboidal regions can be useful for MBDs. Based on these point sets, second-, third-, and fourth-order MBDs in our work are constructed using combinations of the following sets: $C(0^k)$, $F(u^k)$, and $S(u^k, v^{k-a})$. Explanation for this notation is provided in Table (3.4)

In this research second- order minimum bias designs (MBDs) are used to obtain RSM metamodels (see table 3.5)

Table 3.4: $S(u^k, v^{k-a})$ design for k= 3 with $a$ = 1

| $x_1 = \pm u$, $x_2 = \pm v$, $x_3 = \pm v$ | | | $x_1 = \pm v$, $x_2 = \pm u$, $x_3 = \pm v$ | | | $x_1 = \pm v$, $x_2 = \pm v$, $x_3 = \pm u$ | | |
|---|---|---|---|---|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | $x_1$ | $x_2$ | $x_3$ | $x_1$ | $x_2$ | $x_3$ |
| -u | -v | -v | -v | -u | -v | -v | -v | -u |
| -u | -v | +v | -v | -u | +v | -v | -v | +u |
| -u | +v | -v | -v | +u | -v | -v | +v | -u |
| -u | +v | +v | -v | +u | +v | -v | +v | +u |
| +u | -v | -v | +v | -u | -v | +v | -v | -u |
| +u | -v | +v | +v | -u | +v | +v | -v | +u |
| +u | +v | -v | +v | +u | -v | +v | +v | -u |
| +u | +v | +v | +v | +u | +v | +v | +v | +u |

Table 3.5: Second-order MBDs

| $S(u^k, v^{k-a})$ | | | |
|---|---|---|---|
| $k$ | $u$ | $v$ | $a$ |
| 2 | 0.418 | 0.759 | 1 |
| 3 | 0.816 | 0.434 | 1 |
| 4 | 0.868 | 0.448 | 1 |
| 5 | 0.913 | 0.460 | 1 |

More information and details can be found in [6]. Illustrative examples are given in the next subsection.

### 3.2.3 Illustrative example

In this section, a second – order MBD and LHC sample with the same number of design points as the MBD are generated for a two-dimensional space to clarify experimental design generation techniques.

Figure (3.3) shows a second -order MBD for two dimensions X1 and X2 generated using $S(u^{a}, v^{k-a})$ as in Table (3.5).

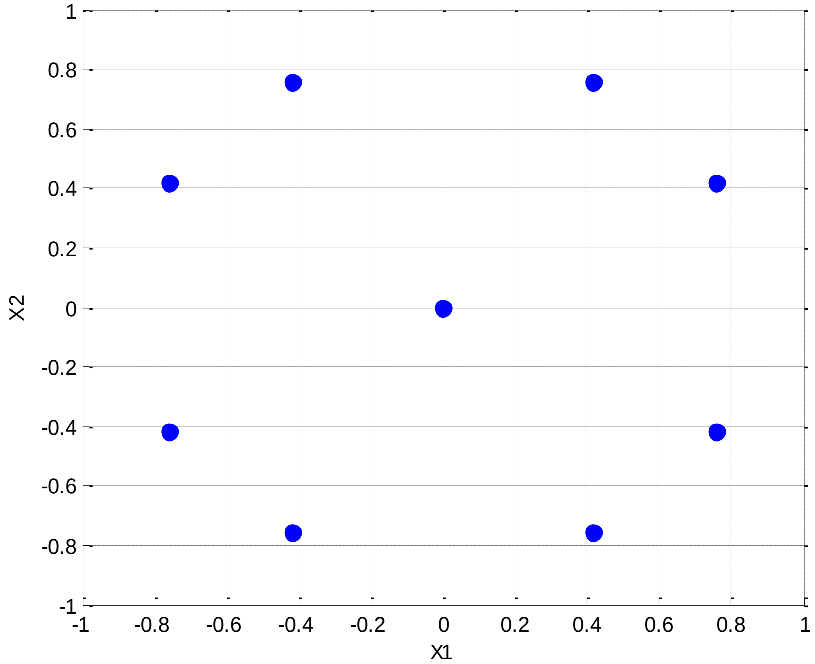


Figure 3.3: second – order MBD

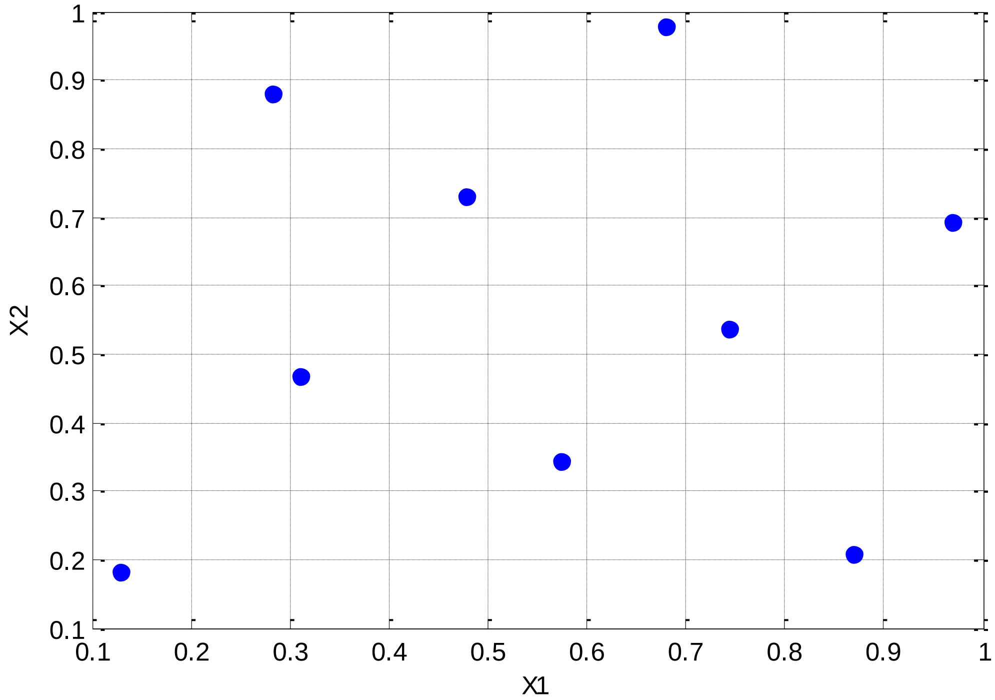


Figure 3.4: second – order LHC (trial 1)

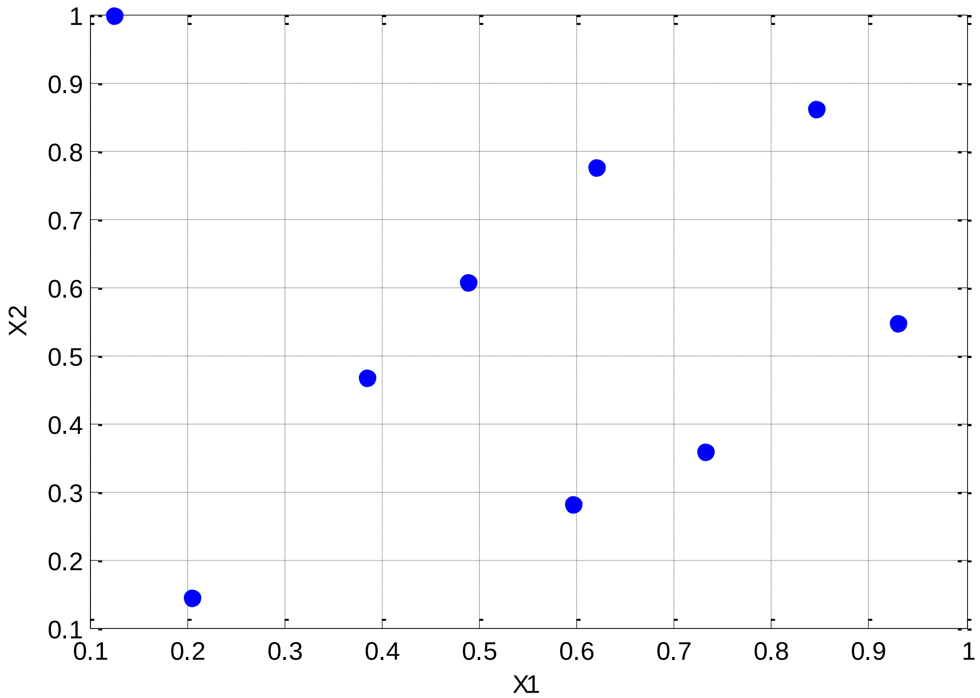


Figure 3.5: second – order LHC (trial 2)

The same number of sample points (i.e., Nine; see Figure 3.3) as the MBD is generated for the LHC sample. Three sampling trials are shown in Figure (3.4), Figure (3.5), and figure (3.6).

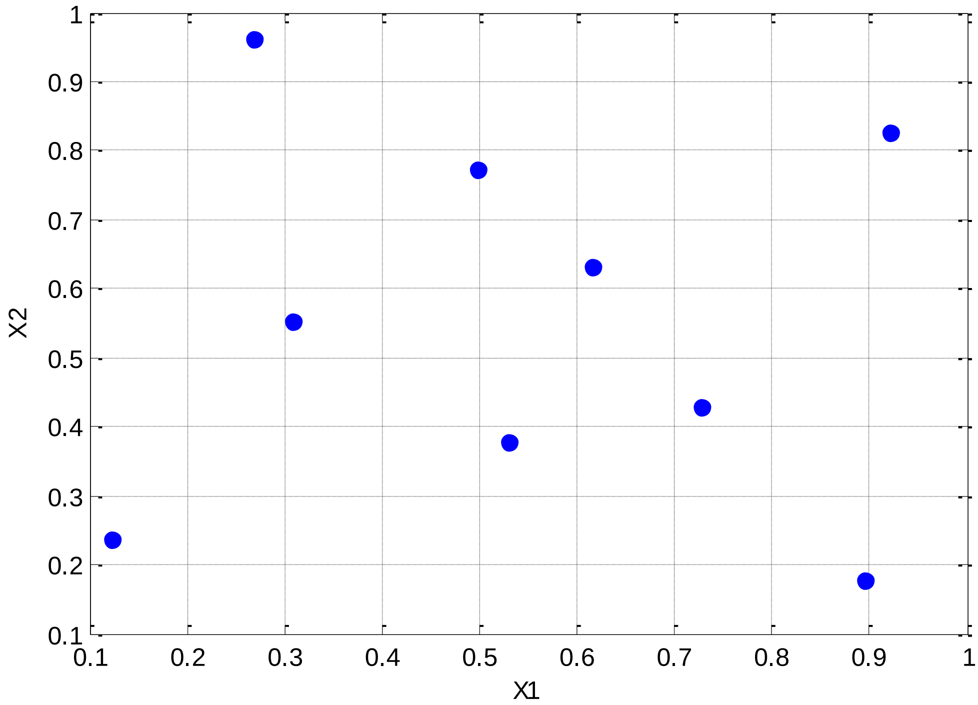


Figure 3.6: second – order LHC (trial 3)

## 3.3 Metamodeling types

As mentioned before, a number of metamodel types are available in the literature; e.g., Kriging [1, 9], Radial Basis Functions (RBF) [1], Response Surface Models (RSM)[4, 17,18], Multivariate Adaptive Regression Splines (MARS)[6] and Artificial Neural Networks (ANN)[6]. In this thesis, we limit our discussion to the most widely used metamodels: Kriging metamodels and RSM.

### 3.3.1 Kriging metamodels

Kriging metamodels were originally developed in geostatistics by the South African engineer Danie Krige [14]. Kriging is an interpolation algorithm for spatial data;

interpolation is a process of finding a value at an unmeasured location from observed data at surrounding locations [1, 8]. A Kriging model estimates the value of the function at a given point by calculating a weighted average of the known values of the function in the surrounding of the point [8].

The Kriging metamodel Ŷ(x) of a response Y(x)is expressed as:

$$\hat{\mathbf{Y}}(\boldsymbol{x}) = \boldsymbol{Y}(\boldsymbol{x}) + \boldsymbol{z}(\boldsymbol{x}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (19)$$

Where ***Y(x)*** is a polynomial of the design variables ***x***, that interpolates the design points. $\boldsymbol{z}(\boldsymbol{x})$ is a Gaussian function that represents the stochastic process (realization of a random process) with zero mean and variance $\sigma^2$ [19]. The goal is to determine weights $\lambda_i$ that minimize the variance.

$$COV(z) = \sigma^2 R(x_i, x_j) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

Where R ($x_i$, $x_j$) is the correlation matrix. More information and details about derivations of Kriging metamodel can be found in [17].

In this work, mGstat Toolbox [21] is used to construct the Kriging metamodels. To use the toolbox in MATLAB we simply added the path where mGstat is installed to the MATALB path. The following example is given to facilitate understanding concepts related to Kriging metamodeling. The response $y(x)$ here is a fifth–order polynomial, over $x \in$ [-3, 4], where:

$$\boldsymbol{y(x) = x^5 - 15x^3 + 20x}$$

Five points are selected in the interval [-3, 4] using LHC experimental design (LHC sampling is discussed in section (3.2.1) to estimate the parameters of the Kriging metamodel $\hat{\boldsymbol{y}}(\boldsymbol{x})$. Figure (3.7) below shows the Kriging metamodel $\hat{y}(x)$ superimposed on the response y(x)

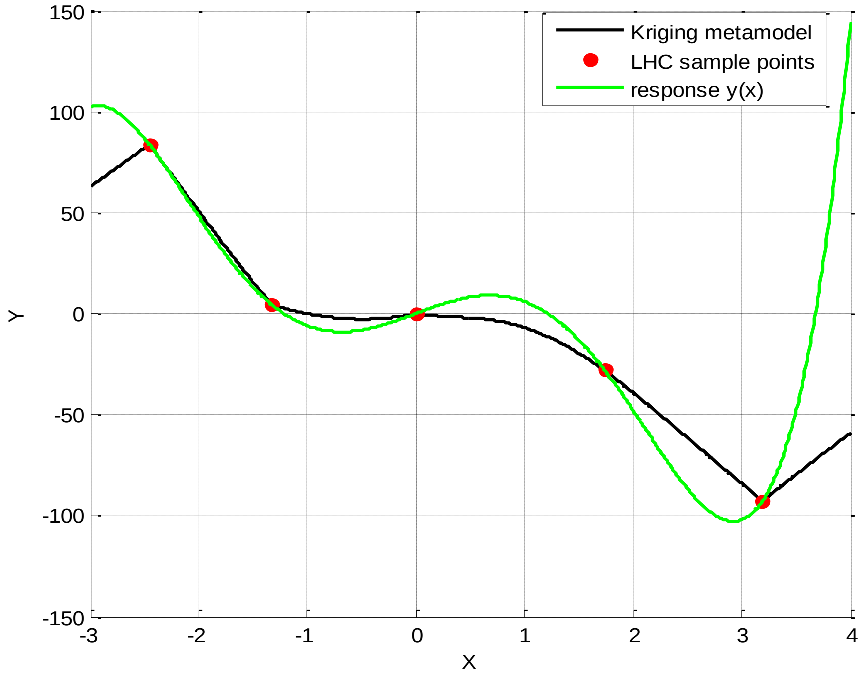


Figure 3.7: response $y(x)$ and Kriging metamodel $\hat{y}(x)$ using 5 samples

### 3.3.2 Response Surface Models (RSM)

The response surface model (RSM) is a polynomial of a response as a function of the various inputs (design variables). It is a parametric regression model, which means that it uses experimental design points to estimate unknown parameters of the polynomial [17, 18].

As it is well known, the behavior of the response surface model depends on the order of polynomial; so the chosen order of the RSM polynomial is important for the accuracy of the metamodel [8].

A second order RSM $\hat{Y}(x)$ in $k$–design variables is expressed as:

$$\hat{Y}(x) = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{i \geq j}^{k} \beta_{ij} x_i x_j \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

$\beta_0$ , $\beta_i$ and $\beta_{ij}$, are the coefficients (parameters) of the RSM, and $x_i$ refers to one of the k design variables.

# CHAPTER IV: METAMODELING OF ANALOG ICs: METHODOLOGY, RESULTS, AND DISCUSSION

In this chapter, Kriging metamodel with Latin hypercube (LHC) samples, and RSMs with MBDs are applied to different analog electronic circuit performance parameters. These circuits range from a simple filter to an operational amplifier - the most widely used analog IC.

## 4.1 Introduction

As mentioned previously, this research focuses on investigating the differences between the two metamodeling techniques of Kriging with LHC sampling and RSM with MBDs. The objective is to find which technique is most accurate for analog integrated circuit design. The statistic used in this work to measure accuracy of metamodeling techniques is the root-mean-square-error (RMSE), which is the most widely used statistic in metamodel validation.

This chapter is organized as follows. Section (4.2) outlines the methodology in this work. In Section (4.3), this research methodology is used to generate performance parameters of a range of analog circuits. Finally, discussion of the metamodeling results is presented in Section (4.4).

## 4.2 Research methodology

Figure (4.1) shows the steps that are used in this work to generate and compare Kriging and RSM metamodels, using LHC and MBD sampling methods respectively.

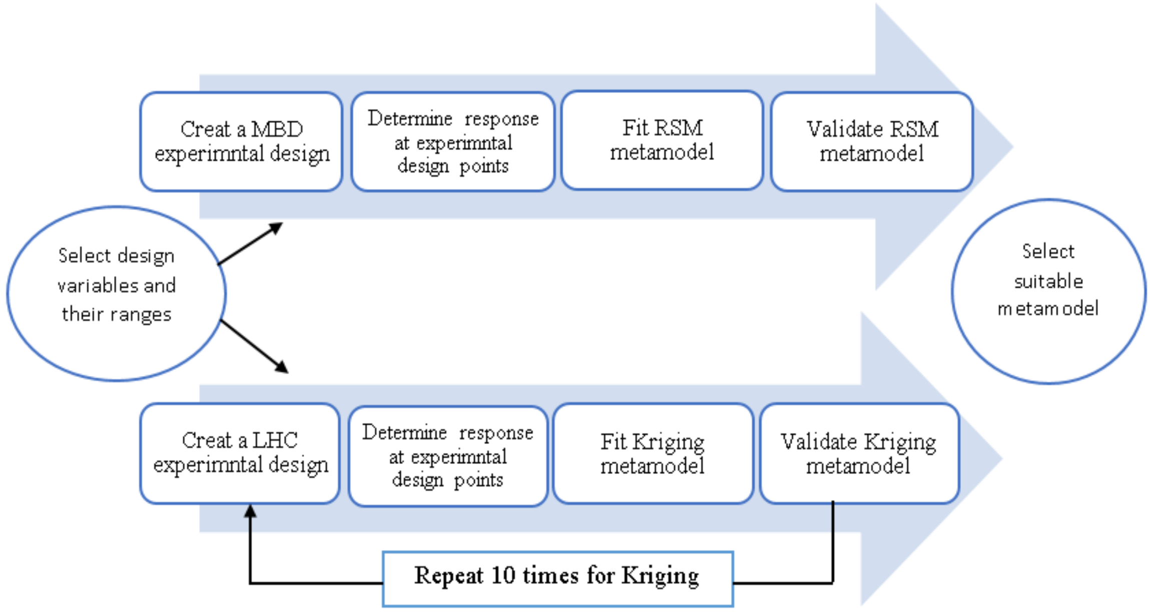


Figure 4.1: Research methodology used for performance parameter metamodeling of electronic circuits.

The metamodeling procedure starts by selecting the design variables as well as their ranges; these are usually provided by the user. In the second phase, two types of computer experimental designs, LHC sampling and MBDs, that were described earlier, respectively, are used to generate the samples points that are used to fit the two types of metamodels: Kriging and RSM. Finally, the metamodel validation phase is conducted by calculating (RMSE) in order to compare the accuracy of the results of the two-metamodeling activities.

## 4.3 Experimental results

The research methodology outlined in the previous section is applied to compare RSM and Kriging metamodels for performance parameters of various electronic circuits, including a simple passive filter, a BJT amplifier, a MOSFET amplifier, and an operational amplifier IC.

### 4.3.1 Passive filter

The performance parameters of the passive low-pass filter shown in Figure (4.2) are modeled using the methodology outlined in Section (4.2). Metamodels for the midband ratio $\zeta = V_{out}/V_{in}$ and the bandwidth (BW) are derived as a function of the five design variables R1, R2, R3, R4 and C.

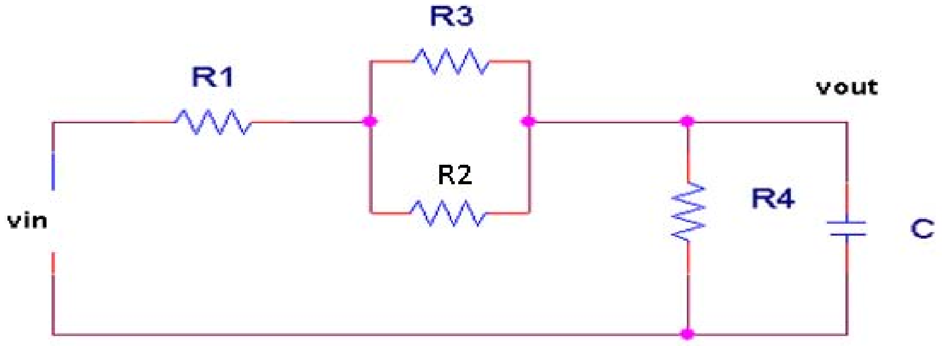


Figure 4.2:Passive filter.

The design variables ranges are given in Table 4.1

Tabel 4.1: design variable ranges for passive filter

| Design variable | Min. value | Max. value | Unit |
|---|---|---|---|
| R1 | 20 | 200 | KΩ |
| R2 | 10 | 100 | KΩ |
| R3 | 10 | 100 | KΩ |
| R4 | 10 | 100 | KΩ |
| C | 1 | 10 | μf |

Firstly, the circuit is modeled using a quadratic RSM in five dimensions. The second-order MBD in Table (3.5) having 161 sample points is used to determine RSM metamodels for responses $\zeta$ and BW. A LHC experimental design is then generated having the same number of sample points as the MBD to fit the Kriging metamodel, LHC sampling is repeated to generate 10 different sampling trials with their corresponding Kriging metamodels. RMSEs for these metamodels are given in Table (4.2), along with NRMSEs, (the errors normalized to the RSME of RSM metamodel).

Table 4.2: Errors values of RSM and Kriging metamodels of ζ

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 0.014 | 1 | 0.160 | 11.4286 |
| | 2 | 0.159 | 11.3571 |
| | 3 | 0.160 | 11.4286 |
| | 4 | 0.161 | 11.5000 |
| | 5 | 0.159 | 11.3571 |
| | 6 | 0.159 | 11.3571 |
| | 7 | 0.160 | 11.4286 |
| | 8 | 0.161 | 11.5000 |
| | 9 | 0.160 | 11.4286 |
| | 10 | 0.159 | 11.3571 |

The normalized errors NRMSEs in Table (4.2) are plotted in Figure (4.3).

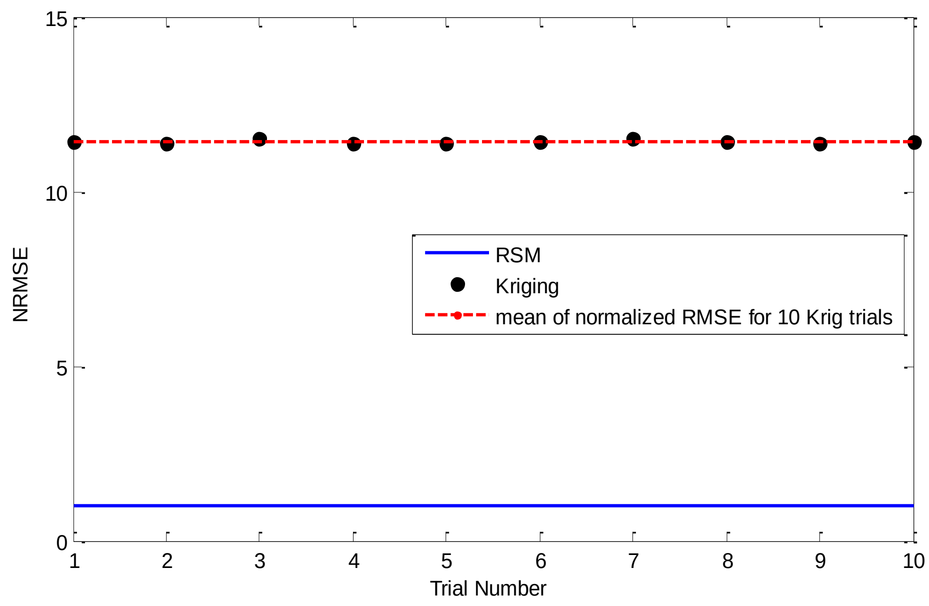


Figure 4.3: NRMSE of ζ for passive filter.

The average RMSE for the response ζ of the filter circuit in Figure (4.3) for the 10 LHC trials for the Kriging metamodel is 0.1598, while the RMSE for the RSM metamodel is 0.014. This means that RMSE for Kriging metamodel is approximately equal to 11X RMSE for RSM. Note that the greater RMSE the worse is the model.

RMSEs for the two metamodels for bandwidth (BW) of the passive filter are given in Table (4.3).

Table 4.3: Errors values of RSM and Kriging metamodels for *BW*

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 0.591 | 1 | 1.378 | 2.3316 |
| | 2 | 1.374 | 2.3249 |
| | 3 | 1.381 | 2.3367 |
| | 4 | 1.379 | 2.3333 |
| | 5 | 1.377 | 2.3299 |
| | 6 | 1.383 | 2.3401 |
| | 7 | 1.374 | 2.3249 |
| | 8 | 1.383 | 2.3401 |
| | 9 | 1.387 | 2.3469 |
| | 10 | 1.385 | 2.3435 |

The normalized RMSEs in Table (4.3) are plotted in Figure (4.4).

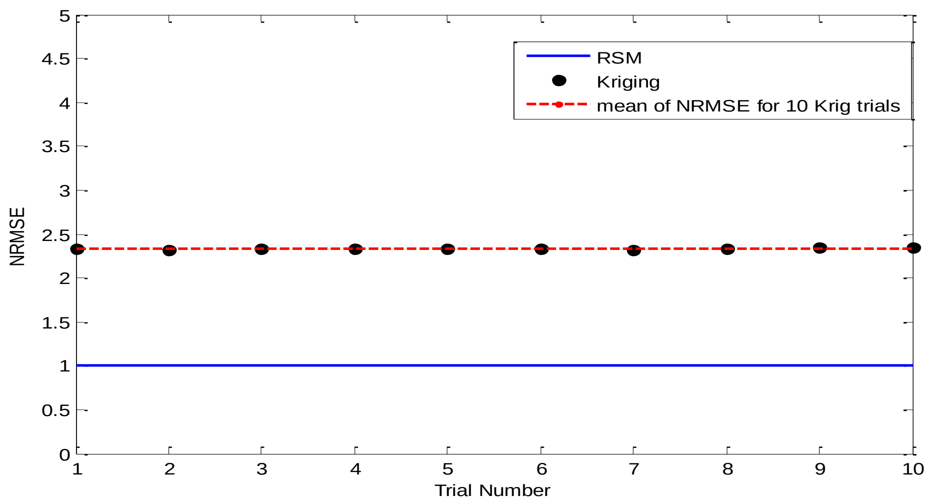


Figure 4.4: NRMSEs for *BW*

The average RMSE for the filter circuit in Figure (4.4) for the 10 LHC trials of the Kriging metamodel is 1.3801, while the RMSE for the RSM metamodel is 0.591. This means that RMSE for Kriging metamodel is approximately equal to 2.33X RMSE of RSM.

### 4.3.2 BJT amplifier

Performance parameters of the BJT amplifier shown in Figure (4.5) are modeled using the methodology outlined in Section (4.2). Metamodels are derived for the midband gain $A = V_{out}/V_{in}$, the lower cutoff frequency $f_l$, and the upper cutoff frequency $f_H$.

The design variables ranges are given in Table (4.4).

Table 4.4: design variables ranges for BJT amplifier

| Design variable | Min. value | Max. value | Unit |
|---|---|---|---|
| C1 | 1 | 10 | µf |
| C2 | 1 | 10 | pf |
| R | 1 | 10 | KΩ |
| I | 1 | 10 | mA |

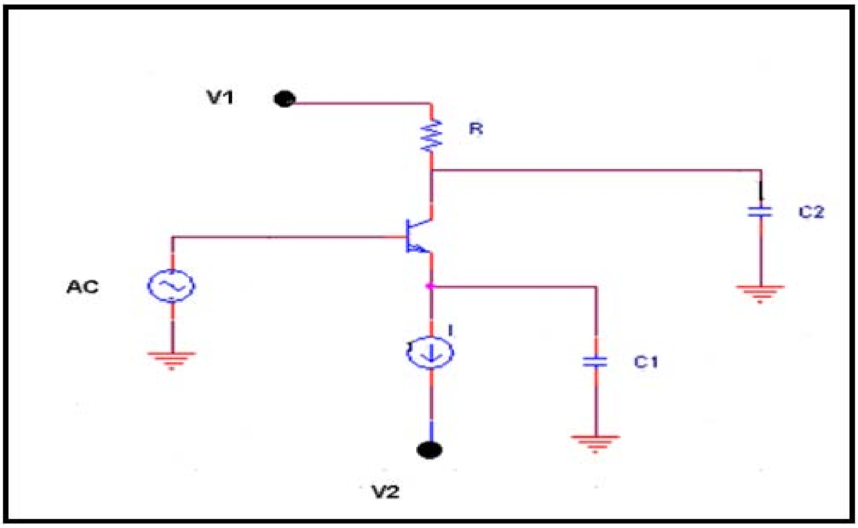


Figure 4.5: BJT amplifier

Firstly, the circuit performance parameters are modeled using quadratic RSMs in four dimensions. The second-order MBD in Table (3.5) having 65 sample points is used to determine responses *A*, $f_1$, and $f_H$ . LHC experimental design is then generated having the same number of sample points as the MBD to fit the Kriging metamodel; the Kriging metamodel activity is repeated using 10 LHC sampling trials. RMSEs and NRMSEs for these metamodels are given in Table (4.5).

Table 4.5: Errors values of RSM and Kriging metamodels for *A*

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 95.78 | 1 | 113.804 | 1.1882 |
| | 2 | 111.366 | 1.1627 |
| | 3 | 111.510 | 1.1642 |
| | 4 | 113.829 | 1.1884 |
| | 5 | 111.274 | 1.1618 |
| | 6 | 111.765 | 1.1669 |
| | 7 | 111.524 | 1.1644 |
| | 8 | 112.317 | 1.1727 |
| | 9 | 111.309 | 1.1621 |
| | 10 | 110.611 | 1.1548 |

The normalized RMSEs in Table (4.5)are plotted in Figure (4.6).

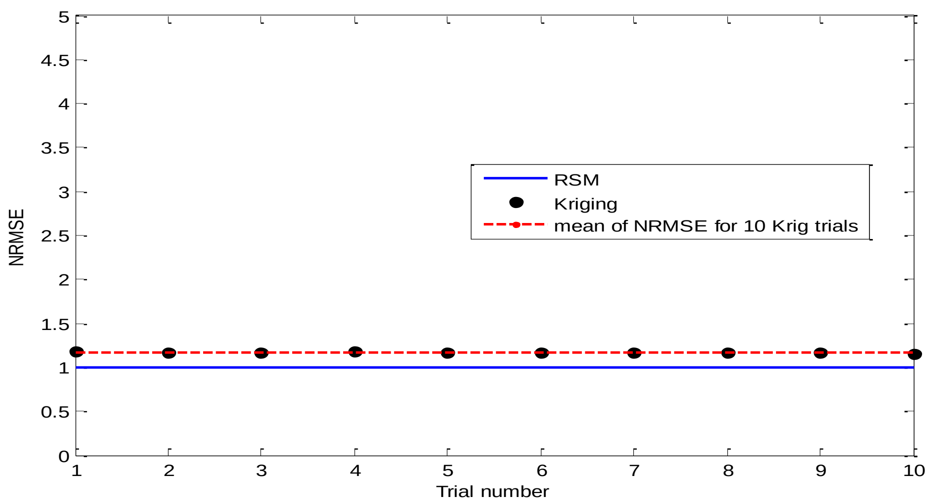


Figure 4.6: NRMSEs for *A* for BJT amplifier

From these results, RMSE for Kriging metamodel for the gain $A$ is higher than the RSM metamodel by approximately 16%.

The previous activities are repeated for generating quadratic RSMs combined with MBD and Kriging metamodels combined with LHC for the lower cutoff frequency $f_l$ and upper cutoff frequency $f_H$. RMSEs and NRMSEs for metamodels for $f_l$ are given in Table (4.6).

Table 4.6: Errors values of RSM and Kriging metamodels for $f_l$

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 28036 | 1 | 35364 | 1.2614 |
| | 2 | 35613 | 1.2703 |
| | 3 | 35604 | 1.2699 |
| | 4 | 35201 | 1.2556 |
| | 5 | 35782 | 1.2763 |
| | 6 | 35609 | 1.2701 |
| | 7 | 35443 | 1.2642 |
| | 8 | 35271 | 1.2581 |
| | 9 | 35358 | 1.2612 |
| | 10 | 110.611 | 1.1548 |

The normalized errors NRMSEs in Table (4.6) are plotted in Figure (4.7).

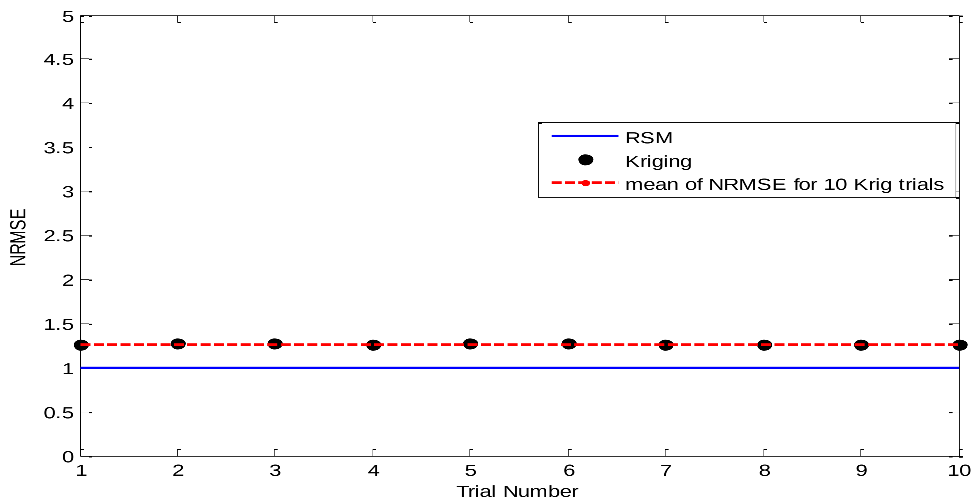


Figure 4.7: NRMSEs of $f_l$ for BJT amplifier

From these results, RMSE for the Kriging metamodel for the lower cutoff frequency $f_l$ is higher than the RSM metamodel by 26%.

The same procedure is repeated for the upper cutoff frequency $f_H$ of the BJT amplifier. RMSEs for the two metamodels are given in the Table (4.7).

Table 4.7: Errors values of RSM and Kriging metamodels for $f_H$

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 33864 | 1 | 41305 | 1.2197 |
| | 2 | 41689 | 1.2311 |
| | 3 | 41395 | 1.2224 |
| | 4 | 41783 | 1.2338 |
| | 5 | 41588 | 1.2281 |
| | 6 | 41442 | 1.2238 |
| | 7 | 42000 | 1.2403 |
| | 8 | 41701 | 1.2314 |
| | 9 | 41993 | 1.2400 |
| | 10 | 41578 | 1.2278 |

The normalized errors NRMSEs in Table (4.7) are plotted in Figure (4.8).

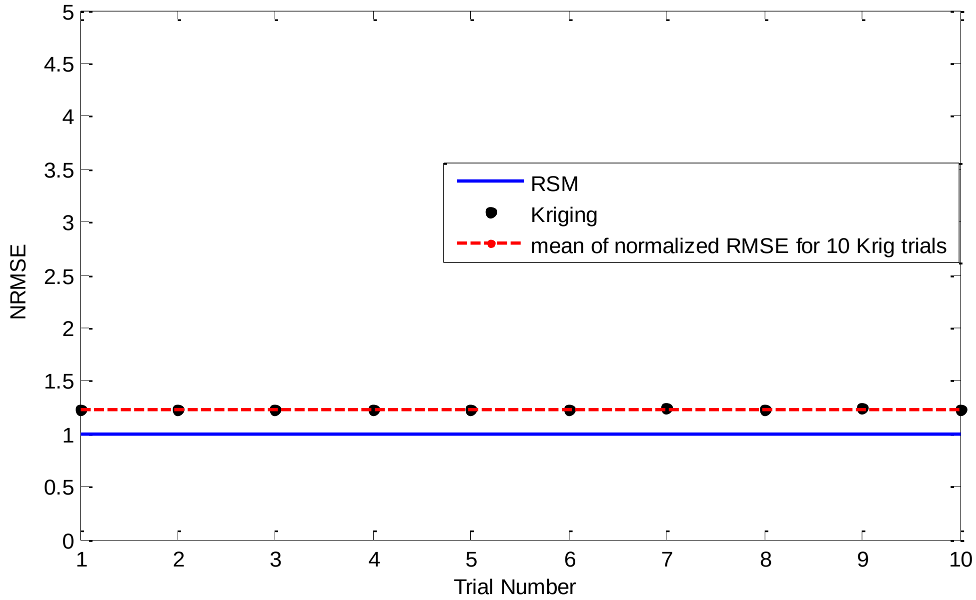


Figure 4.8: NRMSEs of $f_H$ for BJT amplifier

From these results, RMSE for the Kriging metamodel for the upper cutoff frequency $f_l$ is higher than the RSM metamodel by 23%.

### 4.3.3 MOSFET amplifier.

The performance parameters of the MOSFET amplifier shown in Figure (4.9) are modeled using quadratic RSM and Kriging metamodels. For the midband gain $A = V_{out}/V_{in}$ metamodels are derived as functions of the two design variables W1 and W2. (the width of the two MOSFETS M1 and M2, respectively).

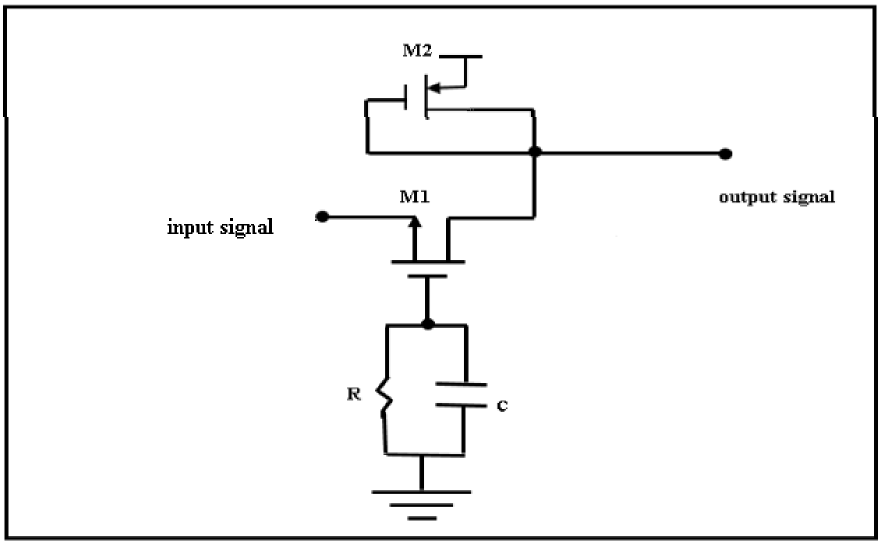


Figure 4.9: MOSFET amplifier.

The design variables ranges are given in Table (4.8).

Table 4.8: design variables ranges for MOSFET amplifier

| Design variable | Min. value | Max. value | Unit |
|---|---|---|---|
| W1 | 2 | 200 | µm |
| W2 | 2 | 200 | µm |
| R | 10 | 100 | KΩ |
| C | 1 | 10 | Pf |

The circuit is modeled using a quadratic RSM in four dimensions. The second-order MBD in Table (3.5) having 65 sample points is used to generate the metamodel for $A$. A LHC experimental design is then generated having the same number of sample points as the MBD to fit the Kriging metamodel, the Kriging metamodel activity is repeated using 10 LHC sampling trials. RMSEs and NRMSEs for these metamodels are given in Table (4.9).

Table 4.9: Errors values of RSM and Kriging metamodels for *A*

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 0.338 | 1 | 0.528 | 1.5621 |
| | 2 | 0.529 | 1.5651 |
| | 3 | 0.531 | 1.5710 |
| | 4 | 0.536 | 1.5858 |
| | 5 | 0.530 | 1.5680 |
| | 6 | 0.529 | 1.5651 |
| | 7 | 0.531 | 1.5710 |
| | 8 | 0.529 | 1.5651 |
| | 9 | 0.528 | 1.5621 |
| | 10 | 0.528 | 1.5621 |

The normalized errors NRMSEs in Table (4.9) are plotted in Figure (4.10).

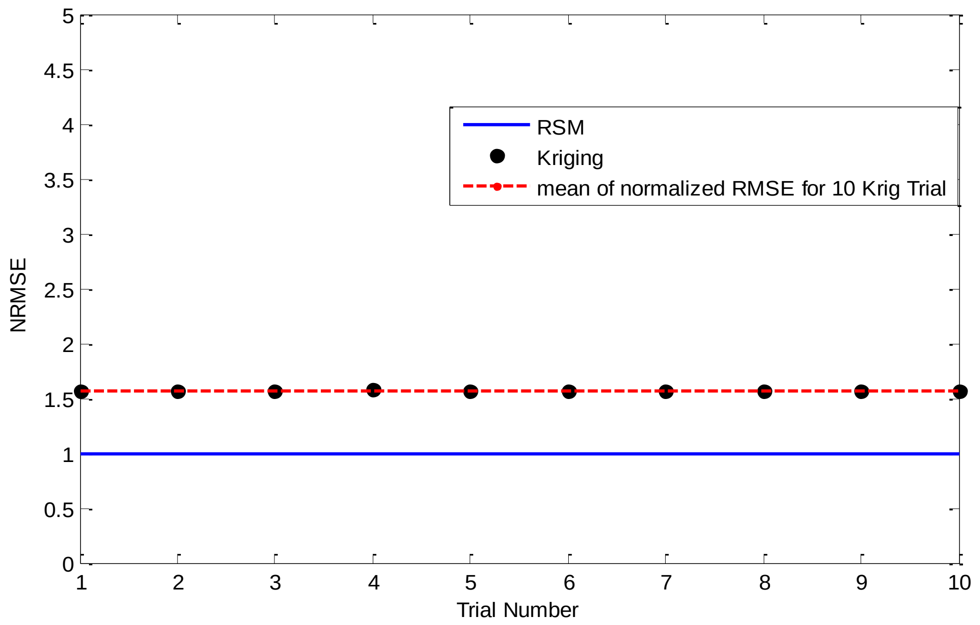


Figure 4.10: NRMSEs of *A* for MOSFET amplifier

From these results, RMSE for the Kriging metamodel for the midband gain $A$ is higher than the RSM metamodel by 56%.

### 4.3.4 Operational amplifier

Performance parameters of the operational amplifier shown in Figure (4.11) are modeled using quadratic RSM and Kriging metamodels. The performance parameters modeled are the midband gain $A = Vout/Vin$ the common- mode- rejection ratio (CMRR) and the power dissipation (P). There are seven design variables W1, W3, W5, W6, W7 W8, and $I_{bias}$ for the corresponding MOSFETs in Figure (4.11).

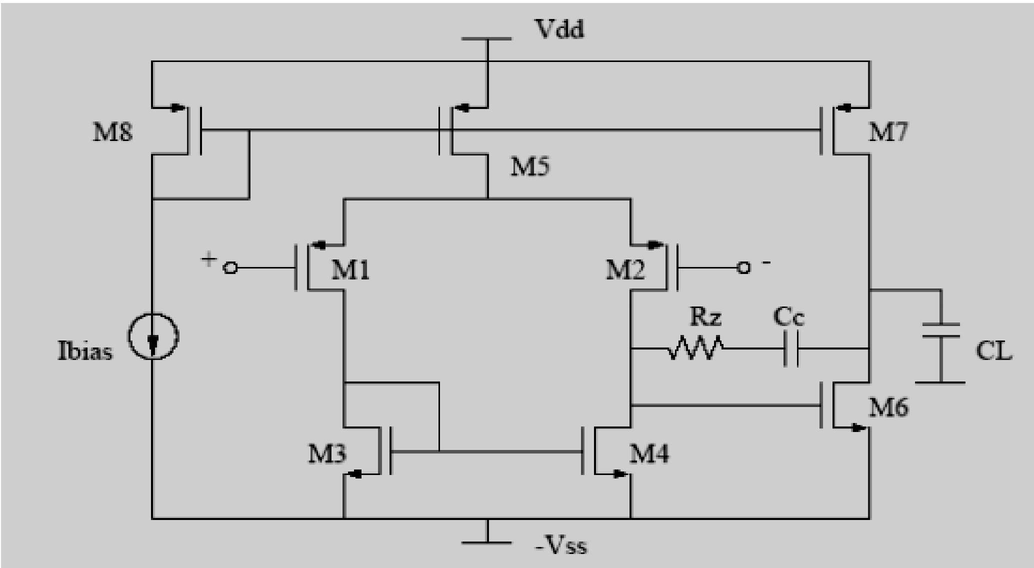


Figure 4.11: operational amplifier

The design variables ranges are given in Table (4.10).

Table 4.10:design variable ranges for Op-amp

| Design variable | Min. value | Max. value | Unit |
|---|---|---|---|
| W1 | 10 | 100 | µm |
| W3 | 10 | 100 | µm |
| W5 | 50 | 100 | µm |
| W6 | 200 | 300 | µm |
| W7 | 100 | 200 | µm |
| W8 | 10 | 100 | µm |
| $I_{bias}$. | 5 | 25 | µA |

Firstly, the circuit is modeled using quadratic RSMs in seven dimensions. The second-order MBD in Table (3.5) having 552 sample points is used to generate RSM metamodel for *A*, *CMMR* and *P*. A LHC experimental design is then generated having the same number of sample points as the MBD to fit the Kriging metamodel. The Kriging metamodel activity is repeated using 10 LHC sampling trials. RMSEs and NRMSEs for these metamodels for *A* are given in Table (4.11).

Table 4.11: Error values of RSM and Kriging metamodels for *A*

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 148212 | 1 | 764042 | 5.1551 |
| | 2 | 764201 | 5.1561 |
| | 3 | 763987 | 5.1547 |
| | 4 | 764061 | 5.1552 |
| | 5 | 764158 | 5.1558 |
| | 6 | 764003 | 5.1548 |
| | 7 | 764261 | 5.1565 |
| | 8 | 764297 | 5.1568 |
| | 9 | 764351 | 5.1571 |
| | 10 | 764146 | 5.1558 |

The normalized errors NRMSEs in Table (4.11) are plotted in Figure (4.12).

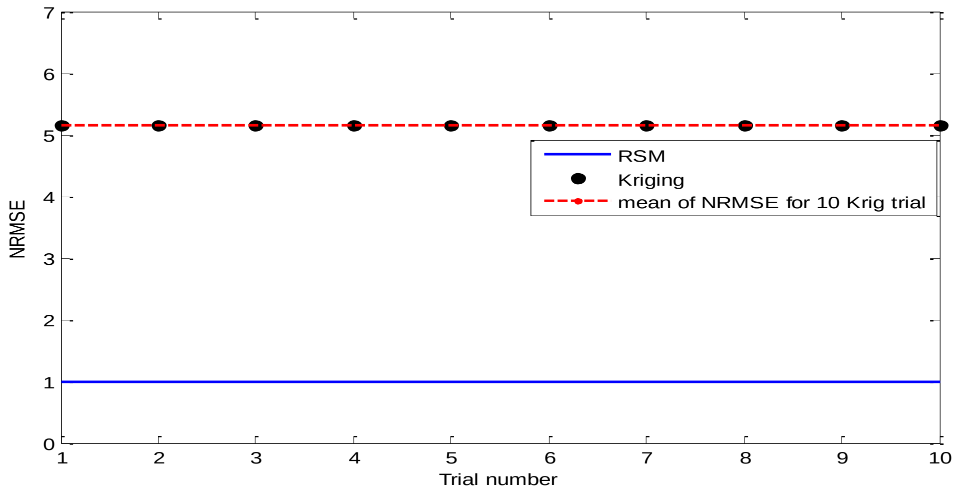


Figure 4.12: NRMSEs of *A* for the operational amplifier

From these results, RMSE for the Kriging metamodel for the midband gain $A$ is approximately equal to 5.15X RMSE for RSM.

RMSEs for metamodels for $CMRR$ of the op-amp are given in Table (4.12).

Table 4.12: Error values of RSM and Kriging metamodels for $CMRR$

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 304125 | 1 | 1366395 | 4.4929 |
| | 2 | 1366416 | 4.4929 |
| | 3 | 1366175 | 4.4921 |
| | 4 | 1366311 | 4.4926 |
| | 5 | 1366370 | 4.4928 |
| | 6 | 1366420 | 4.4930 |
| | 7 | 1366196 | 4.4922 |
| | 8 | 1366234 | 4.4923 |
| | 9 | 1366402 | 4.4929 |
| | 10 | 1366323 | 4.4926 |

The normalized errors NRMSEs in Table (4.12) are plotted in Figure (4.13).

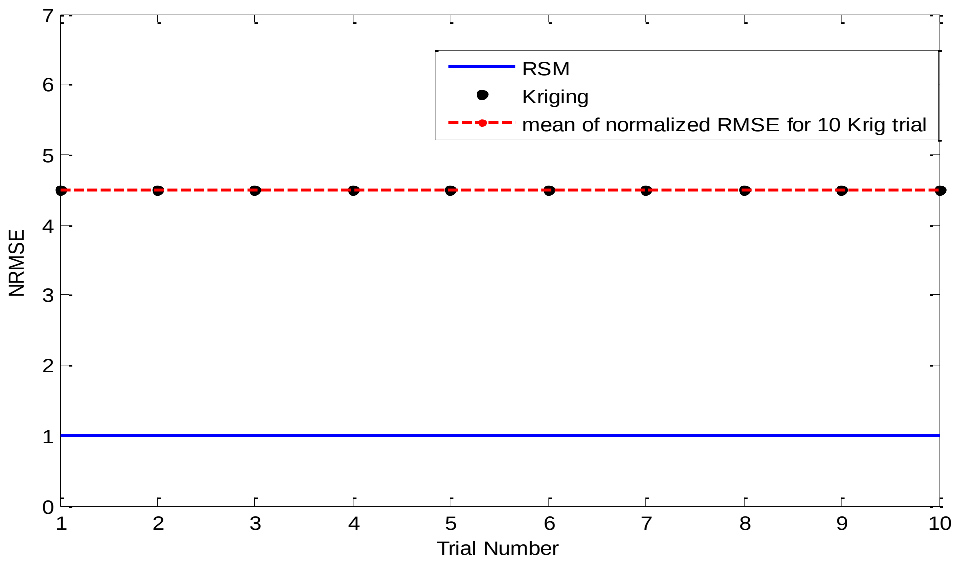


Figure 4.13: NRMSEs of CMRR for the operational amplifier

From these results, RMSE for the Kriging metamodel for the common-mode-rejection-ratio *CMRR* is approximately equal to 4.5X RMSE for RSM.

RMSEs of two metamodels for P of the op-amp are given in Table (4.13).

Table 4.13: RMSE values for RSM and Kriging metamodels for P

| RMSE for RSM metamodel | Kriging | | |
|---|---|---|---|
| | Trial number | RMSE | NRMSE |
| 126.699 | 1 | 389.023 | 3.0705 |
| | 2 | 388.892 | 3.0694 |
| | 3 | 389.261 | 3.0723 |
| | 4 | 389.025 | 3.0705 |
| | 5 | 388.997 | 3.0702 |
| | 6 | 389.318 | 3.0728 |
| | 7 | 389.230 | 3.0721 |
| | 8 | 389.001 | 3.0703 |
| | 9 | 389.225 | 3.0720 |
| | 10 | 389.542 | 3.0745 |

The normalized RMSEs in Table (4.13) are plotted in Figure (4.14).

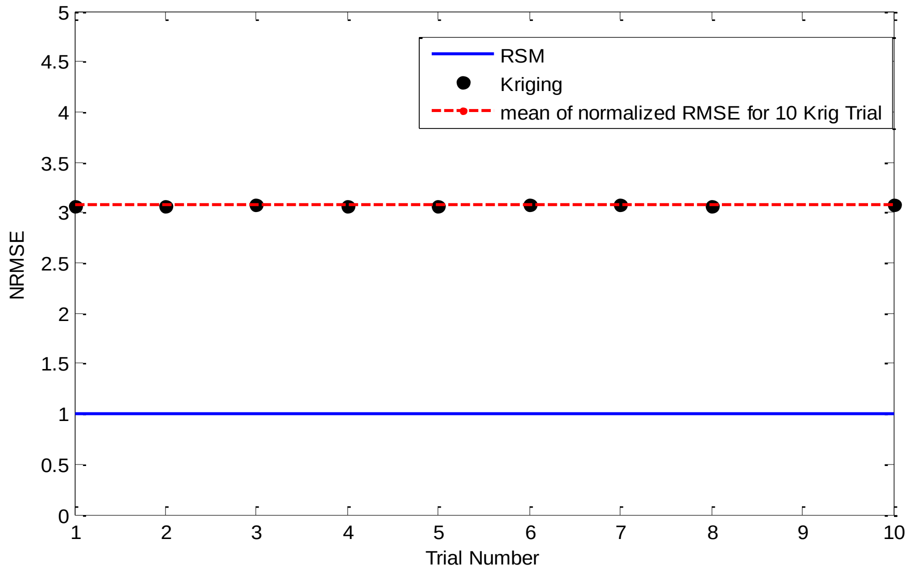


Figure 4.14: NRMSEs for P for operational amplifier

From these results, RMSE for the Kriging metamodel for the common-mode-rejection-ratio $CMRR$ is approximately equal to 3X RMSE for RSM.

## 4.4 Discussion

Based on the results shown in the previous section, it is clear that the maximum difference value of NRMSE between RSM and Kriging metamodels is in the passive filter for the midband ratio ζ, the error ratio is 1:11.5. On the other hand, the minimum difference value of NRMSE between RSM and Kriging metamodels is in BJT amplifier for the gain *A*, with an error ratio of 1:1.16.

So, it is clear that the MBD sampling combined with RSM is superior to LHC combined with Kriging in all presented examples.

# CONCLUSIONS AND FUTURE WORK

Metamodeling is used as a way to reduce computational efforts in engineering system design while still revealing accurate information about the behavior of the simulation model of the engineering system.

In this thesis, two types of metamodeling techniques - Kriging metamodels generated using LHC sampling, and response surface models (RSMs) based on minimum bias designs (MBDs) are studied. Based on Google Scholar search results as discussed in Chapter one, the former metamodeling technique (i.e. Kriging metamodel with LHC designs) is more recent and gaining popularity in the literature, as compared to the classical technique (i.e. RSM metamodels with MBDs) which – as it seems – is phasing out.

Both metamodeling techniques are applied in this work to different analog integrated circuits. The results show that using minimum bias designs MBDs to obtain RSM metamodels improves accuracy in relation to Kriging metamodels with LHCs.

These results should direct the science community to use RSM with MBDs more frequently in metamodeling and not to ignore it.

## FUTUTE WORK

To improve the accuracy of the metamodel, piecewise RSM metamodels with MBDs sampling method may be used, whereby the design variables space may be partitioned based on validation results for a coarse global (covering the whole design variables space) metamodel.

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# ملخص

**الخطيب، هدى تركي. حول استخدام التصاميم التجريبية في نمذجة الدوائر الالكترونية التناظرية المتكاملة. رسالة ماجستير بجامعة اليرموك. 2017. (المشرف الرئيسي: د. سامي الحمدان، المشرف الثانوي: أ. د. حسام حمد).**


**النموذج البديل "metamodel " هو نموذج للنموذج. وتسمى عملية توليد او تصميم مثل هذه النماذج نمذجة "metamodeling". وهكذا فان النموذج البديل هو تجريد مبسط للنموذج الأصلي الذي يجعل المحاكاة الحاسوبية المعقدة للنموذج الأصلي أبسط ولكن أسرع مع الحفاظ على دقة مقبولة. أصبحت النماذج البديلة تستخدم على نطاق واسع في التخصصات الهندسية والعلمية. كما يتطلب النموذج الناجح اختيار دقيق للتصميمات التجريبية "experimental design" المناسبة. والتصميم التجريبي هو عباره عن مجموعه من القيم المتغيرة (المدخلات) التي يتم استخدامها لبناء النموذج البديل. في هذا البحث تم مناقشة اثنين من أكثر النماذج البديلة شهرة. الأول هو "classical response surfaces" والأكثر حداثة هو "Kriging models ". مع اثنين من التصاميم التجريبية "Latin hypercube (LHC) sampling" و " minimum (MBDs) bias design". وتطبق كل طريقة في هذا العمل من أجل وضع نماذج لمخرجات الدوائر الإلكترونية التناظرية. وتبين نتائج هذا البحث أن استخدام "classical response surfaces" مع "(MBDs)" يعطي نتائج أفضل وأدق من النماذج الأكثر شيوعا "Kriging models" مع " (LHC)". كما يترتب بناء على هذا البحث توجيه المجتمعات البحثية للدوائر المتكاملة على اعتماد طريقة "MBD" مع "RSM" في بناء النماذج بشكل أكثر وذلك لثبوت جدواه من حيث دقته في محاكاة النموذج الأصلي.**